ROTATION OF A MAGNETORHEOLOGICAL SUSPENSION IN A CONSTANT MAGNETIC FIELD II. UNBOUNDED AND SEMIBOUNDED CYLINDERS

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The specifics of the hydrodynamics of magnetorheological suspensions (MRS) rotating magnetic field were considered in [1]. It was shown that a torque component directed alo the surface arises as a result of curvature at the free surface. This causes the develop of a secondary flow. The equations of motion and the condition at the free surface have formulated for this flow. The flow has the nature of a boundary layer with two sharply different viscosity characteristics: the radial η_1 and the axial η_2 viscosity ($\eta_1 \gg \eta_2$). The flow intensity depends on the η_1 and η_2 values as well as the magnetic stress $\tilde{\sigma}_{\varphi r}^{M}$.

The values of η_1, η_2, and $\tilde{\sigma}_{\varphi r}^{M}$ are determined by the MRS microstructure. In a magnet field, the dispersed phase particles aggregate in ellipsoidal clusters, which are equally oriented in relation to the field. Therefore, the MRS microstructure is characterized by quantities: the form parameter r_e (ratio of the ellipsoid axes) and the orientation angl [1]. Two equations are necessary for their determination. One of them, which relates th quantities, is obtained by equating the magnetic and the hydrodynamic torques (relationsh (4) in [1]). In order to derive the second equation, we consider the forces acting on an ellipsoid rotating at the angular velocity Ω in the liquid.

The stresses at the surface of the ellipsoid, in a coordinate system bound to it (Fi 1) are given by [2]

$$\sigma_y = -p_0 P \frac{y}{b^2} + \frac{8\eta_0 P}{ab^2} G \frac{z}{a^2}, \qquad \sigma_z = -p_0 P \frac{z}{a^2} + \frac{8\eta_0 P}{ab^2} G' \frac{y}{b^2},$$

where $P=(y^2/b^4+z^2/a^4)^{-1/2}$ is the distance between the coordinate origin and the tangent plane the point (y, z), p_0 is the hydrostatic pressure, G and G' are coefficients, equal to

$$G = \frac{a^2\Omega}{2(a^2\alpha_0+b^2\beta_0)}, \qquad G' = -\frac{b^2\Omega}{2(a^2\alpha_0+b^2\beta_0)},$$

respectively, and α_0 and β_0 are elliptic integrals.

For the most interesting case of strong magnetic fields, where $r_e=a/b\gg 1$, elliptic integrals permit simple representation, which allows us to write the expressions for the stresses in the following form:

$$\sigma_y = -p_0 P \frac{y}{b^2} + \frac{2\eta_0 P z\Omega}{b^2 \ln r_e},$$
$$\sigma_z = -p_0 P \frac{z}{a^2} - \frac{2\eta_0 P y\Omega}{b^2 \ln r_e}.$$

Analysis of system (2) shows that, in our case ($r_e \gg 1$), the stress $\sigma = \sqrt{\sigma_y^2 + \sigma_z^2}$ reaches its extremum value at the point $z\approx a$; $\tilde{y}=0$.

Equations (2) determine the stress acting at the poles of a cluster, which causes the cutoff of "excess" particles. The cutoff of particles is counteracted by the magnetic fo acting between them. As was shown in [3], the magnetic force is equal to

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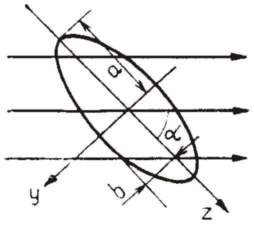

Fig. 1. Statement of the problem.

$$F_{\mathrm{mag}} \approx \frac{B_a^2}{\mu_0 \mu_a} \frac{2\pi R^2}{3\Phi} \frac{R}{\Delta}, \tag{3}$$

where B_a and μ_a are the magnetic field induction in the cluster and its permeability, respectively, Φ is the fill factor of the cluster with respect to particles ($\Phi \approx 0.5$), R is the particle radius, and Δ is the minimum layer thickness.

For sufficiently large values of r_e, the demagnetization factor along the major axis vanishes, and, therefore $B_a \approx \mu_0 \mu_a H_e$ and

$$F_{\mathrm{mag}} \approx \mu_0 \mu_a H_e^2 \frac{2\pi R^2}{3\Phi} \frac{R}{\Delta}. \tag{4}$$

It also follows from (2) that the hydrodynamic force acting on a single particle is equal to

$$F_{\mathrm{hyd}} \approx 2\pi R^2 \eta_0 \Omega r_e^2 / \ln r_e. \tag{5}$$

In the state of equilibrium,

$$F_{\mathrm{hyd}} \approx k F_{\mathrm{mag}}. \tag{6}$$

The quantity k in (5) is the effective coefficient accounting for a somewhat elevated magnetic force (expression (4) is satisfied exactly only if $\Delta \ll R$), a lower hydrodynamic force (the effective particle area in relationship (5) is assumed to be equal to πR^2), and, what is most important, the existence of a sufficiently "thick" liquid interlayer between particles, which keeps the friction coefficient at a very low level.

Substitution of the magnetic and the hydrodynamic forces in (6) results in the relationship

$$\frac{r_e^2}{\ln r_e} \frac{1}{S_1} = k \frac{\mu_a}{\chi_a} \frac{R}{\Delta} \frac{1}{3\Phi},$$

where we have introduced the parameter $S_1 = \mu_0 \chi_a H^2 / (\eta_0 \Omega)$ – the ratio of the magnetic torque to the hydrodynamic torque.

It is evident that the complex $r_e^2 / (\ln r_e S_1)$ is independent of the process parameters and that it is determined only by the type of the dispersed medium (the characteristic particle size, the permeability of clusters, and the character of particle packing in them). Since, for $r_e \gg 1$, the quasisolid rotational motion of the medium is equivalent to shear flow, the value of this complex can be obtained by analyzing the experimental data on Couette flow [3]. This yields

$$r_e^2 \approx 0.08 \ln r_e S_1. \tag{7}$$

Equation (7) constitutes the sought dependence of the form parameter r_e on the dimensionless number S_1. Together with relationship (4) from [1], it determines the MRS microstructure, and thereby the values of η_1, η_2, and $\sigma_{\varphi r}^M$.

We shall first analyze the simple case where the free-surface effect is absent. Let us consider the hydrodynamics of a MRS which fills the clearance between infinite concentric cylindrical surfaces with the radii r_1 and r_2 ($r_2 > r_1$). The angular velocity of one of them is assigned, while that of the other is unrestricted. The magnetic field is perpendicular t the cylinder axis. Similar problems for magnetic liquids (ML) have been analyzed in many papers, of which we shall mention [4-7].

For this flow, the equation of motion has the following form:

$$\frac{d}{dr}\left[r^3\frac{d}{dr}\left(\frac{v}{r}\right)\right]=0. \tag{8}$$

We shall first consider the case where the angular velocity of the outer surface is assigned:

$$v|_{r=r_2}=-\Omega r_2. \tag{9}$$

Considering the condition of absence of shearing stresses at the inner surface and relationships (6) from [1], we arrive at the expression

$$\left[\eta_1 r\frac{d}{dr}\left(\frac{v}{r}\right)+\bar{\sigma}_{\varphi r}{}^{M}\right]\Bigg|_{r=r_1}=0 \quad (\bar{\sigma}_{\varphi r}{}^{M}<0). \tag{10}$$

The solution of problem (8)-(10) provides the distribution of the angular velocity of MRS:

$$\omega=-\Omega+\frac{\bar{\sigma}_{\varphi r}{}^{M}r_1{}^2}{2\eta_1 r^2}\left[1-\left(\frac{r}{r_1}\right)^2\right].$$

The magnetic field accelerates the rotation of MRS. The increment of the angular velocity of the inner cylinder is equal to

$$\frac{\Delta\omega}{\Omega}=\frac{\omega}{\Omega}\Big|_{r=r_1}-1=-\frac{\bar{\sigma}_{\varphi r}{}^{M}}{2\eta_1\Omega}\left[1-\left(\frac{r_1}{r_2}\right)^2\right]. \tag{11}$$

Let us analyze the asymptotic behavior of the quantity $\Delta\omega$ in strong fields. Relationships (9)-(11) from [1] indicate that, under these conditions, the viscosity η_1 tends to the value

$$\eta_1\approx\eta_0\varphi_a r_e{}^2/[8(\ln r_e-0.8)]. \tag{12}$$

The magnetic stresses are determined on the basis of (8) and (2) [1]:

$$|\bar{\sigma}_{\varphi r}{}^{M}|\approx\mu_0 H^2(\pi/2-\alpha^1)\varphi_a\chi_a{}^2/(\chi_a+2). \tag{13}$$

As follows from Eq. (4) [1], the orientation angle in strong fields is equal to

$$\alpha'=\pi/2-\frac{2r_e{}^2}{\ln r_e}\,\frac{1}{S_1}\,\frac{\chi_a+2}{\chi_a}. \tag{14}$$

As a result, we determine from (11)-(14) the sought expression for the asymptotic behavior of excess of the inner cylinder's angular velocity:

$$\left(\frac{\Delta\omega}{\Omega}\right)_{\max}=8\left[1-\left(\frac{r_1}{r_2}\right)^2\right](1-0.8/\ln r_e). \tag{15}$$

The value of the form factor r_e in fairly strong fields ($H \sim 10^4$ A/m) reaches 40...60 [3]. For moderately concentrated media ($\varphi\approx 0.1$), this can double the angular velocity if $r_1/r_2 = 0.83$ (such a ratio of radii was realized in ML experiments [7]).

If the angular velocity of the inner cylinder (Ω) is assigned, the velocity distributio in the clearance is given by

$$\omega=-\Omega+\frac{\bar{\sigma}_{\varphi r}{}^{M}}{2\eta_1\Omega}\left(\frac{r_2}{r}\right)^2\left[1-\left(\frac{r}{r_1}\right)^2\right]$$

The liquid rotates more slowly than the inner cylinder. The outer cylinder also lags behind the inner cylinder. In strong fields, this velocity "deficit" is equal to

$$\left(\frac{\Delta\omega}{\Omega}\right)_{max}=8\left[\left(\frac{r_2}{r_1}\right)^2-1\right](1-0.8/\ln r_e). \tag{16}$$

For the sake of comparison, we shall briefly describe the results for ML's. A higher rotational speed of the inner cylinder in comparison with that of the outer one is also observed for these liquids. This occurs due to the orienting effect of the magnetic field on the dispersed phase elements. The particles and the liquid have different angular velocities, which triggers the rotational viscosity mechanism. This effect is much more strongly pronounced in the case of MRS's due to the intensive structurization. The developing clusters are no longer disoriented as a result of Brownian movement. This causes a considerable increase in the magnetic torque.

We shall provide a quantitative comparison. The maximum angular velocity excess of the inner cylinder occurring in the absence of friction in the bearings is, in the case of a ML, equal to [7]

$$\left(\frac{\Delta\omega}{\Omega}\right)_{ml}=\frac{s[1-(r_1/r_2)^2]}{1+s(r_1/r_2)^2},$$

where $s = \eta_r/\eta$ is the ratio of the rotational viscosity to the shear viscosity; the shear viscosity is equal to $\eta_r = \tau_\perp M_0 H/4$, where $\tau_\perp$ is the relaxation time of the magnetization component perpendicular to the field, and M_0 is the equilibrium magnetization. Generally, the rotational viscosity is determined experimentally [7] because of the difficulties in calculating the value of $\tau_\perp$. The rotational viscosity increases with the degree to which the magnetic component is frozen in and also with a rise in the field strength. Its limiting value is equal to $\eta_r=3\eta\varphi/2$. It occurs for ML's with a frozen-in structure, where rotational diffusion, caused by Brownian movement, is absent. The Brownian movement can be neglected if the Langevin parameter $\xi=mH/kT$ (m is the magnetic moment of a particle) is equal to ~20. For magnetite at room temperature, a field of the order of 10^5 A/m is necessary for this. Under these conditions,

$$\left(\frac{\Delta\omega}{\Omega}\right)_{ml}=\left(\frac{\Delta\omega}{\Omega}\right)_{ml,\ max}=\frac{1.5\varphi[1-(r_1/r_2)^2]}{1+1.5\varphi(r_1/r_2)^2}. \tag{17}$$

For $\varphi=0.1$ and $r_1/r_2 = 0.83$, this yields 4.2%, which is a much lower value than in the case of MRS's. In the experiments described in [7], the excess of the inner cylinder's angular velocity reached ~12%. If friction in the bearings is taken into account, this is realized for $\eta_r \approx 0.5\eta$.

It is interesting to note that, in strong fields, this effect is independent of the dispersed phase concentration for MRS's, in contrast to ML's. This is connected with the fact that both the torque and the viscosity η_1 increase with φ for MRS's. In the case of ML's, the torque increases, while the shear viscosity remains constant.

Thus, comparative analysis indicates that the effect of the magnetic field on the MRS hydrodynamics sharply increases with the formation of immobile, long clusters.

Let us now consider the effect of a free surface. This effect has not been analyzed for ML's. We shall examine the hydrodynamics of a MRS in a rotating cylindrical vessel. The equation and the boundary condition at the free surface for the developing secondary flow were provided in [1]:

$$\frac{1}{\bar{r}^2}\frac{\partial}{\partial\bar{r}}\left[\bar{r}^3\frac{\partial}{\partial\bar{r}}\left(\frac{\bar{v}}{\bar{r}}\right)\right]+\frac{\partial^2\bar{v}}{\partial\bar{z}^2}=0, \tag{18}$$

$$-\beta\bar{r}^2\frac{\partial}{\partial\bar{r}}\left(\frac{\bar{v}}{\bar{r}}\right)+\frac{\partial\bar{v}}{\partial\bar{z}}=\bar{r}\quad \text{for}\quad \bar{z}=0. \tag{19}$$

where

$$\beta=\sqrt{\frac{\eta_1}{\eta_2}\,\frac{\Omega^2R}{g}},\quad \bar{v}=-\frac{\Omega r+v_\varphi}{V},\quad \bar{r}=r/R,$$
$$\bar{z}=\sqrt{\eta_1/\eta_2}\,z/R,\quad V=-\bar{\sigma}_{r\varphi}^{0}\Omega^2R^2/(\sqrt{\eta_1\eta_2}\,g). \tag{20}$$

These relationships are supplemented with the conditions for secondary flow attenuation within the liquid, limitation of the velocity at the axis, and adhesion at the side surfaces of the vessel:

$$\bar{v}=0 \text{ for } \bar{z}\to-\infty, \quad \bar{v}=0 \text{ for } \bar{r}=1. \qquad (21), (22)$$

The solution of Eq. (18) that satisfies condition (21) is given by

$$\bar{v}=\sum_{n=1}^{\infty} A_n \exp(\sqrt{\lambda_n}\,\bar{z}) I_1(\sqrt{\lambda_n}\,\bar{r}), \qquad (23)$$

where $I_1(x)$ is a first-order Bessel function of the first kind. From condition (23) follows the equation for eigenvalues of the problem:

$$I_1(\sqrt{\lambda_n})=0,$$

the solution of which has been tabulated [8].

We use condition (19) for determining the unknown coefficients A_1, A_2, ... By expanding the left-hand and the right-hand sides of relationship (19) in a Fourier series with respect to Bessel functions and considering that, for Bessel functions, the derivatives can be expressed in terms of the functions themselves, we reduce (19) to the following form:

$$-\beta\sum_{n=1}^{\infty} A_n[\bar{r}\sqrt{\lambda_n} I_0(\sqrt{\lambda_n}\bar{r})-2I_1(\sqrt{\lambda_n}\bar{r})]+\sum_{n=1}^{\infty} A_n\sqrt{\lambda_n} I_1(\sqrt{\lambda_n}\bar{r})= \\ =\sum_{n=1}^{\infty}\frac{2}{\sqrt{\lambda_n} I_2(\sqrt{\lambda_n})} I_1(\sqrt{\lambda_n}\bar{r}). \qquad (24)$$

By expanding $f(\bar{r})=\bar{r}I_0(\sqrt{\lambda_n}\bar{r})$ in a Fourier series with respect to the $I_1(\sqrt{\lambda_n}\bar{r})$ function, we transform relationship (24) in the following manner:

$$\sum_{m=1}^{\infty}\beta I_1(\sqrt{\lambda_m}\bar{r})\left(-\sum_{n=1}^{\infty}A_n\sqrt{\lambda_n}c_{nm}+2A_m\right)+ \\ +\sum_{m=1}^{\infty}A_m\sqrt{\lambda_m}I_1(\sqrt{\lambda_m}\bar{r})=\sum_{m=1}^{\infty}\frac{2}{\sqrt{\lambda_m}I_2(\sqrt{\lambda_m})}I_1(\sqrt{\lambda_m}\bar{r}), \qquad (25)$$

where

$$c_{mn}=\frac{2}{I_2^2(\sqrt{\lambda_m})}\int_0^1 \bar{r}I_0(\sqrt{\lambda_n}\bar{r})I_1(\sqrt{\lambda_m}\bar{r})\,d\bar{r}. \qquad (26)$$

By equating in (26) the coefficients in front of the corresponding terms of the expansion, we obtain for the coefficients A_n an infinite system of linear algebraic equations:

$$A_n(\sqrt{\lambda_n}+2b)-\beta\sum_{m=1}^{\infty}c_{mn}\sqrt{\lambda_m}A_m=\frac{2}{\sqrt{\lambda_n}I_2(\sqrt{\lambda_n})}, \quad n=1,2,\ldots \qquad (27)$$

The system of linear algebraic equations (27) was solved numerically by using the elimination method. The integrals in the expression for c_{mn} were found by numerical integration according to Simpson's rule. The values of the Bessel functions figuring in these integrals were also determined numerically with $\epsilon \sim 10^{-5}$. Substitution of the A_n coefficients in relationship (23) yielded the sought velocity profile. Numerical calculations have shown that it is sufficient to retain 30 terms of the expansion in expression (24) in order to secure a velocity determination accuracy equal to $\epsilon_1 = 10^{-4}$. Therefore, we solved system (27) for 30 equations with respect to the sought values of A_n for a 30 × 30 c_{mn}. At the same time, the equation

$$\bar{Q}=2\pi\sum_{n=1}^{\infty}A_n\exp(\sqrt{\lambda_n}\bar{z})\int_0^1\bar{r}I_1(\sqrt{\lambda_n}\bar{r})\,d\bar{r}$$

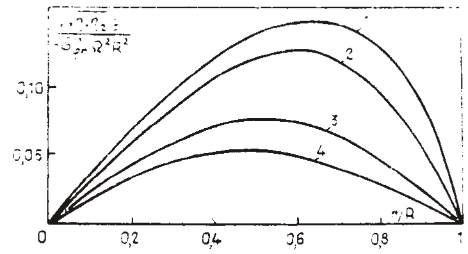

Fig. 2. Dependence of the relative velocity of secondary flow on r/R for the following values of the β parameter: 1) 0.5; 2) 1; 3) 3; 4) 5.

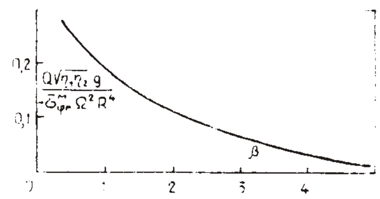

Fig. 3. Relative discharge as a function of the β parameter.

was used for determining the relative discharge. Some of the calculation results are given in Figs. 2-5. The secondary flow intensity is determined by 1) extrinsic – magnetic (H) and hydrodynamic (Ω) – parameters; 2) intrinsic – characteristics of the dispersion medium (η_0) and the dispersed phase (κ_a and φ_a) – parameters; 3) a geometric factor – the vessel radius (R). In order to analyze their effect on the velocity v and the discharge Q, it is necessary to determine the functional relationships between the stress $\bar{\sigma}_{\varphi r}^M$ and the viscosities η_1 and η_2 on the one hand, and the above quantities on the other. Of greatest interest are strong magnetic fields. By using expressions (13) and (14) for $\bar{\sigma}_{\varphi r}^M$ (12) for η_1, the results obtained in [1], and relationship (7), we obtain the following for strong magnetic fields (considering that $\ln r_e \gg 1$):

$$\bar{\sigma}_{\varphi r}^M \approx -0{,}16\mu_0 H^2\chi_a\varphi_a, \quad \eta_1 \approx 0{,}01\mu_0 H^2\chi_a\varphi_a/\Omega, \quad \eta_2=\eta_0. \tag{28}$$

Then, the relationships for the characteristic velocity V, the parameter β, and the scale of the depth reached by the secondary flow $\tilde{z}=R\sqrt{\eta_2/\eta_1}$ are given by

$$V\approx 1{,}6H\Omega^{5/2}R^2\sqrt{\mu_0\varphi_a\chi_a/\eta_0/g}, \quad \beta\approx 0{,}1H\sqrt{\mu_0\varphi_a\Omega R\chi_a/(g\eta_0)}, \quad \tilde{z}=R/10\sqrt{\eta_0\Omega/(\mu_0 H^2\chi_a\varphi_a)}. \tag{29}$$

This makes it possible to estimate (according to Figs. 2-5) the effect of all the assigned quantities on the process characteristics. The secondary flow intensity increases with the field strength. This is caused by the intensification of structurization, which leads to a reduction of the demagnetization factor and a sharp increase in the magnetic torque. In very strong fields, where the degree of disparity between cluster diameters is high, field intensification no longer produces a decrease in the demagnetization factor, and the secondary flow increases in proportion to $\sqrt{H}$. As a result of this, as the field strength increases by a factor of 10 (from β = 0.5 to β = 5), the maximum velocity increases by a factor of less than 4 (Fig. 2). Another fact should be mentioned. In rotational flow of a MRS in the clearance between coaxial cylinders with free solid boundaries, the secondary flow in strong fields is virtually independent of the field strength (Eqs. (15) and (16)). In the presence of a free surface, the effect of H persists even for strong fields. This is connected with the planeness of the flow, for which the effective viscosity is equal to $\sqrt{\eta_1\eta_2}$. Therefore, the characteristic velocity $V\sim -\bar{\sigma}_{\varphi r}^M/\sqrt{\eta_1\eta_2}$ in strong fields is a linear function of

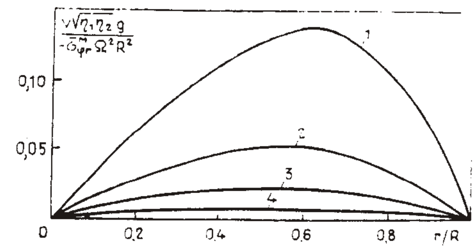

Fig. 4. Secondary flow intensity as a function of the relative distance to the free boundary $\bar{z}$: 1) 0; 2) 0.25; 3) 0.5; 4) 1.

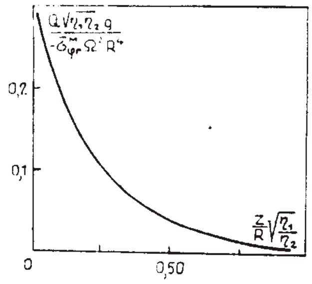

Fig. 5. Variation of the relative discharge along the depth in the liquid.

H according to relationship (29), while, in the case of unbounded cylinders, we have $V \sim -\bar{\sigma}$ $\eta_1 = \text{const}(H)$. This is explained by the ease with which laminar flow of the liquid is established if $\eta_1 \gg \eta_2$.

The curvature radius of the free surface diminishes with an increase in the angular velocity. This brings about an increase in the magnetic torque component oriented in the direction parallel to the MRS surface and an increase in the secondary flow intensity. Moreover, as is indicated by relationship (5), the hydrodynamic stresses which cut off the particles at the cluster poles increase with Ω. This reduces the form factor r_e and, thus, the viscosity η_1, which produces an additional increase in the velocity v. A reduction in magnetic stresses, which could reduce the value of v, has not been observed, since the demnetization factor becomes insensitive to the parameter r_e for sufficiently large values of the latter.

Let us analyze the effect of the intrinsic MRS parameters. According to (28), an increase in the viscosity of the dispersion medium produces an increase in the viscosity η_2 and, as a consequence, a reduction in the value of V and, therefore, in the secondary flow intensity. However, this effect is relatively weak, since the magnetic stress is independ of η_0. This viscosity affects only r_e; the stress $\bar{\sigma}_{\varphi r}{}^{M}$ in strong fields is independent of r_e. As the concentration of the dispersed phase and its magnetic susceptibility increase, there is an increase in both the magnetic stresses and the radial viscosity η_1. However, t axial viscosity η_2 remains constant. Therefore, the characteristic velocity V and the para eter β functionally depend equally on φ_a and χ_a. Considering the results of numerical calculations, we find that this produces both higher velocities (Fig. 2) and higher discharge values (Fig. 3).

With increasing distance from the surface, the secondary flow intensity drops (Figs. 4 and 5). This drop has a strongly pronounced nonlinear character. Thus, the secondary flow is localized in the surface layer, which is also indicated by qualitative analysis.

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