DYNAMICS OF A HETEROGENEOUS MEDIUM WITH SELF-EXCITING INTERNAL OSCILLATIONS WITHIN A MAGNETIC FIELD

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UDC 532.135

Suspensions of particles polarized by external fields (electric or magnetic), exhibiting strong long-range effects, have presently become quite popular. An important property of such media is the possibility of controlling the level of mechanical energy dissipation (effective viscosity) as a consequence of reversible structuring. A number of models has been proposed in which the main role in examination of microhydrodynamics is ascribed to the influence exerted by particle interaction with the external field [1—5]. As the intensity of the shearing flow is increased the structure breaks down into its individual elements (aggregates) whose dimensions depend on the conditions under which the system is deformed. In view of the fact that the shearing flow is "layered," we are dealing here with the motion of one such layer made up of polarized particles entrained by the carrier fluid, relative to some other layer. Since the particles have not been frozen into the layer, additional displacement relative to a viscous carrier medium becomes possible as a consequence of the interaction with the fields on the part of particles from an adjacent layer. Such self-excited motion on the part of particles may serve as an additional source of mechanical energy dissipation. The present study is devoted to the simulation, investigation of and evaluation of the role of this particular mechanism in energy conversion.

One-dimensional motion in a viscous medium of a solitary magnetized spherical particle within a field whose strength is perpendicular to the velocity vector and which changes periodically in space serves as the simplest situation in which this formulation of the problem is realized.

Translated from Magnitnaya Gidrodinamika, No. 2, pp. 27-32, April-June, 1990. Original article submitted March 10, 1989; revision submitted June 22, 1989.

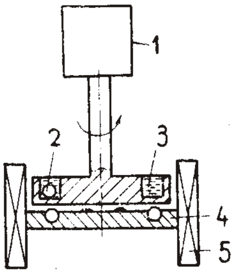

Fig. 1. Diagram of experimental installation: 1) torque meter; 2) test specimen; 3) rectangular annular channels; 4) ring with fixed bearings; 5) solenoid.

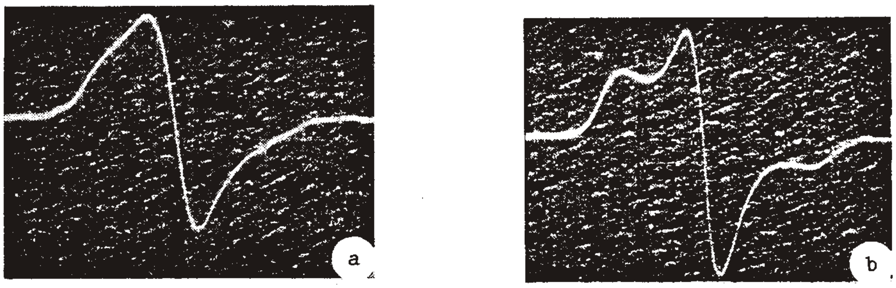

Fig. 2. Signal from induction sensor in the case of a fixed (a) and a free (b) particle.

An annular rectangular channel filled with a viscous fluid, containing a test spherical particle of magnetic materials, is mounted on the measuring unit of a rotation viscosimeter (Fig. 1). At a distance commensurate with the diameter of the particle, underneath the channel, we find a ring with rigidly fixed and uniformly distributed such particles. The ring plane is perpendicular to the rotor shaft. Both the channel and the ring are fabricated out of nonmagnetic material and positioned within a solenoid so as to cause the field force lines to be parallel to the axis of channel rotation.

A microwinding is situated on one of the fixed particles, to function in the role of induction sensor, from whose signals recorded on an oscillograph screen it becomes possible to draw conclusions with regard to the nature of the motion executed by the test particle. The magnetic field of the solenoid is oriented as indicated, it magnetizes both the test particle and the attached bearings, and together these form a magnetic field which periodically varies about the circumference. The rotational frequency of the rotor is varied in these experiments, as is the field intensity of the solenoid, the number of nonmoving particles, and the viscosity of the fluid in the channel. We measured the rotor torque and the velocity of the test particle relative to the channel walls: the pulses from the induction sensor were recorded.

These experiments showed that in the absence of a field the particle moves at an angular velocity equal to the rotational velocity of the channel; with an increase in field intensity the particle decelerates all the way to complete stoppage when $H = H_{cr}'$; with a further decrease in the intensity of the magnetic field the particle begins to move when $H = H_{cr}'' < H_{cr}'$ (the process of stopping and starting exhibits a hysteresis nature). When the test particle is attached to the channel wall, the signal from the induction sensor is in the shape of a symmetrical sinusoid (Fig. 2a), i.e., the work performed by the forces of magnetic particle interaction with the spheres fixed within the ring is equal to zero; the particle moving freely induces a periodic sign-changing nonsymmetric signal, which indicates pulsations in its velocity (Fig. 2b); the braking torque on the cuvette, owing to hydrodynamic losses ascribed to particle streamlining, increases with a rise in field intensity and reaches its maximum when the particle comes to a halt.

The equation for the one-dimensional motion of the test particle in a viscous medium has the form

$$mR\dot{\omega} - F_1[R(\omega_0 - \omega)] = F_2(\varphi). \quad (1)$$

Here φ is the angular coordinate; m, mass of the particle; ω_0, angular rotational velocity of the cuvette; $\omega = d\varphi/dt$; R, radius of the circle described by the rotation of the bearing center; F_1, the force of viscous resistance; F_2, an external periodic force; we assume that the fluid within the cuvette rotates in quasihard fashion and that the motion of the particle introduces no perturbation into this flow.

We will initially examine the model problem in which the particle has to overcome a potential barrier of constant height, i.e., with a stepwise change in the functions

$$F_2(\varphi)=\begin{cases}0 & \text{with } n\Phi\leqslant\varphi<n\Phi+\varphi_0;\\ -\bar{F} & \text{with } n\Phi+\varphi_0\leqslant\varphi<(n+1)\Phi,\end{cases}\qquad(2)$$

where Φ is the period of the function $F_2(\varphi)$, and where we have the linear resistance law $F_1[R(\omega_0-\omega)]=kR(\omega_0-\omega)$. Of greatest interest is the steady-state periodic motion whose characteristics are determined by the relationship between the competing forces, i.e., the viscous forces (wherein consideration is given to the law governing the velocity of the relative particle motion) and magnetic forces. Transition to dimensionless variables on the basis of formulas

$$\varphi_*=\varphi/\Phi;\quad \tau=tk/m;\quad \Omega=\omega_0 m/(k\Phi);\quad C=\varphi_0/\Phi;$$
$$a=\bar{F}m/[k^2\Phi R-\omega m/(k\Phi)]$$

and the subsequent solution of the dynamic problem (1) and (2) lead to the following relationships (within a single period)

$$v-\omega_*+\Omega\ln[(\Omega-v)/(\Omega-\omega_*)]-\varphi_*=0;\quad 0\leqslant\varphi_*<C;$$
$$\omega_{*c}-\omega_*-\varphi_*+C+a\ln[(a+\omega_*)/(a+\omega_{*c})]=0;\quad C\leqslant\varphi_*<1,\qquad(3)$$

where $\omega_*=d\varphi_*/d\tau$ represents the dimensionless angular velocity of motion; v is the minimum velocity (for $\varphi_*=1$); ω_{*c} is the velocity when $\varphi_*=C$. The quantities v and ω_{*c} are determined from the solution for the system of equations which is derived from (3) when $\varphi_*=C$ for the former relationship and when $\varphi_*=1$ for the latter relationship.

This problem contains the independent quantities Ω, a, and C, which determine the dynamics of particle motion. Of considerable importance is the limit case of overcoming the barrier, as $v\to 0$. Then there exists a relationship between Ω, a, and C. We will subsequently examine the relationship of the width of the carrier [the quantity $(1-C)$] for specified values of Ω and a. Analysis of the solution shows that there exists a C_{min} such that it is possible to overcome the barrier only when $C\geq C_{min}$. The article is decelerated only when $a\geq 0$. When $a=0$, $C_{min}=1-\Omega(1-e^{-1/\Omega})$. It is interesting to note that in this case the time required for the bearing to pass the segment without a field (the acceleration segment) is independent of the length of the segment (the quantity C), i.e., $\tau_C=1/\Omega$; in this case, with an increase in the height of the barrier the length of the segment increases, i.e., $\partial C_{min}/\partial a>0$. For the maximum possible magnitudes of the decelerating field ($a\to\infty$) the minimum length of the acceleration segment is equal to $C_{min}=\Omega[\ln(1-t_\Omega)^{-1}-t_\Omega]$, where the parameter $t_\Omega=\omega_{*c}$, i.e., is the solution of the equation

$$\Omega\ln(1-t_\Omega)+\Omega\xi\ln(1+t_\Omega/\xi)+1=0\qquad(4)$$

(the quantity $\xi=a/\Omega$). The physical significance of t_Ω is the fact that it represents a dimensionless velocity normalized to Ω, at the end of the acceleration segment.

Equation (4) has a single root. Expanding $\ln(1+t_\Omega/\xi)$ into a series over the small parameter t_Ω/ξ ($t_\Omega/\xi\ll 1$, since $t_\Omega<1$), we obtain the equation for the determination of t_Ω:

$$1+\Omega\ln(1-t_\Omega)+\Omega t_\Omega-\Omega t_\Omega^2/2\xi=0,\qquad(5)$$

whose solution yields $C_{min}=1-\Omega t_\Omega^2/2\xi$. The work of the field on the braking segment is equal to $\Omega t_\Omega^2/2$. Relationship (5) shows that no matter how large the barrier (braking force a), provided that its width is less than $\Omega t_\Omega^2/2a$, the particle will be able to overcome the barrier.

If the width of the barrier is greater than critical, the particle is decelerated to complete cessation of movement at the boundary $\varphi_*=C$. Once the quantity C_{max}, dependent on a and Ω, is exceeded, it is even possible that after complete cessation of movement the particle will begin to move in the opposite direction.

Numerical calculations have been carried out to determine the relationship between the magnitude of the critical barrier and its width and the velocity of the fluid (by critical we mean the maximum barrier which the particle can surmount). Some of these calculation results can be seen in Fig. 3. The maximum velocity which the bearing acquires depends strongly on the length of the acceleration segment, but only for the comparatively short wavelengths C. The

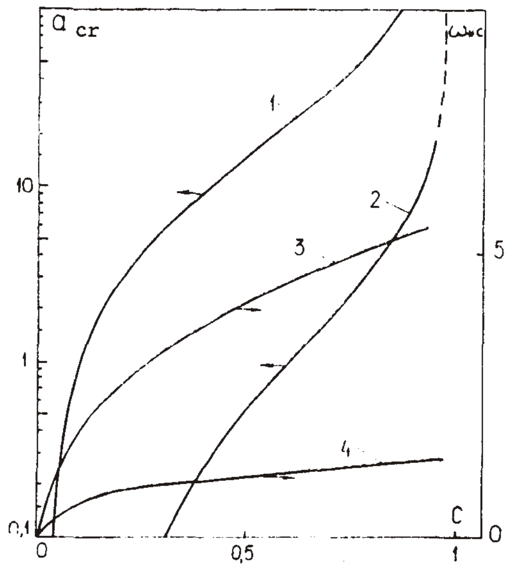

Fig. 3. Critical barrier a_{cr} (1, 2) and maximum velocity ω_{*c} (3, 4) as functions of the width of the decelerating-field region when Ω = 20 (1, 3) and 2 (2, 4).

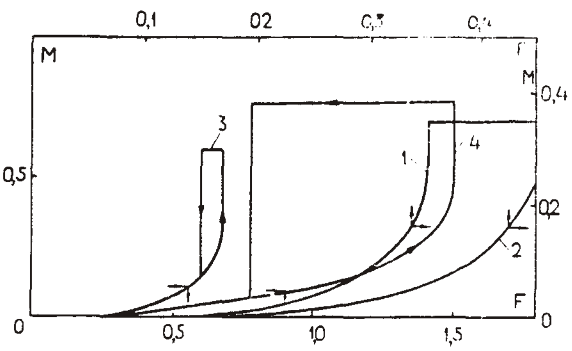

Fig. 4. The effect of the magnetic-field force F on the magnitude of the additional moment M when Ω = 0.0667 (1), 0.133 (2), 0.267 (3), and 0.533 (4).

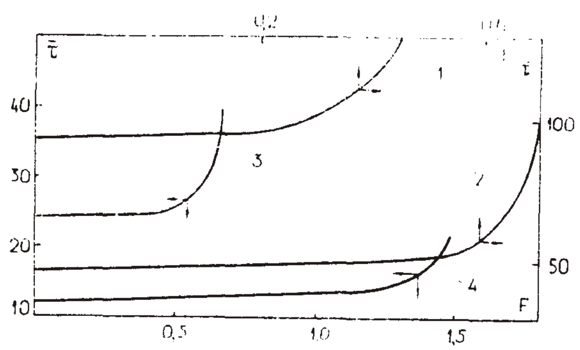

Fig. 5. Particle reversal time $\overline{\tau}$ as a function of magnetic-field force for the same values of Ω as in Fig. 4.

critical-barrier height a_{cr} function undergoes particularly marked change for values of C close to unity. Here we observe agreement between the numerical calculations and the asymptotic formulas. These graphs enable us to find a solution even for the case in which the particle is in motion within a viscous fluid whose flow velocity is ω_0, as well as in the presence of segments in which the direction of the force f is accelerational in certain segments and decelerational in others (the amplitude of the force is assumed to be identical in each of these segments). The critical values of f_{cr} and ω_{0cr} are associated with a and Ω by the relationships

$$f_{cr}=(\Omega+a)/2;\quad \omega_{0cr}=(\Omega-a)/2. \tag{6}$$

Thus, with a fixed value for ω the particle separates when $f' = \omega_{0cr}$, while it comes to a complete stop when $f'' = f_{cr}$, which is determined from formula (6). It follows from the graph that with C = 0.5 (the acceleration and deceleration segments are equal) and with $\Omega = 20$, the quantity $a_{cr} = 14.5$, from which $f_{cr} = 17.25$, $\omega_{0cr} = 3.75$. Thus, particle separation occurs at $f' = 3.75$, while it stops completely when $f'' = 17.25$, i.e., when we have hysteresis.

Most realistic is the case in which the external force changes in accordance with the harmonic law $F_2(\varphi) = F_0 \sin(N\varphi)$, where N represents the number of lower bearings. With a function such as $F_2(\varphi)$ the analytical solution of Eq. (1) becomes impossible. For a qualitative study we will construct the phase portrait of the equation of motion, namely:

$$\frac{d\omega_*}{d\varphi_*}=\frac{\Omega-\omega_*+f_0\sin(N\varphi)}{\omega_*}.$$

When $\Omega > f_0$, the integral curves lie in the region $\omega_* > 0$, i.e., cessation of motion for the particles is impossible. With $\Omega < f_0$, singular points appear within the function. In each period $\varphi \in [2\pi n, 2\pi(n+1)]$ we find two. The type of the first one $\varphi_0 \in [(2n+1)\pi, (2n+3/2)\pi]$ of them depends on the f_0 and Ω relationship: 1) with $\Omega < f_0 < \Omega\sqrt{1+(1/4\Omega)^2}$, a stable point; 2) $f_0 = \Omega\sqrt{1+(1/4\Omega)^2}$, a subcritical point; 3) $f_0 > \Omega\sqrt{1+(1/4\Omega)^2}$, the focus. The second point $\varphi_0 \in [(2n+3/2)\pi, 2(n+1)\pi]$ is a saddle point. The initial conditions govern whether or not an image reaches one or another singular point. An analysis of the phase portrait shows that in this case hysteresis is observed: the maximum force of retention for the nonmoving bearing is $f' = \Omega$, while periodic trajectories are possible when $f > \Omega$.

In order to obtain quantitative characteristics of the process of bearing motion we have to solve Eq. (1) under the specific laws governing the forces F_1 and F_2, dependent on the conditions of bearing motion and the law governing the change in the magnetic field and also on the magnetic characteristics of the bearing. In view of the fact that the hydrodynamic pattern of the compressed streamlined bearing is quite complex (the lateral cross section of the channel exhibits a diameter that is on the order of that of the bearing), the form of the functional relationship for $F_1(v)$ (v is the relative speed of the bearing) is determined experimentally. With this purpose in mind, a rather strong magnetic field is applied to the "bearing-fluid" system, rotating at an angular velocity ω_0, and this field would bring the bearing to a stop, forcing it to rotate in place. We measured the additional torque M_1 at the axis of the cuvette. Subsequently, in order to eliminate the forces of interaction between the bearing and the bottom of the cuvette, we conducted an analogous experiment in the absence of fluid and we determined the moment M_2. The sought force of resistance is $F_1(\Omega R_1) = (M_1 - M_2)/R_1$ (R_1 is the distance from the axis of the cuvette to the center of the bearing). By conducting these studies for a variety of values we were able to determine the relationship

$$F_1(v)=k_1v^p\,(k_1=0{,}03\ \text{N}\cdot\text{sec}^{\,p}\cdot\text{m}^{-p},\ p=0{,}39). \tag{7}$$

The form of the function $F_2(\varphi)$ was also found experimentally. The bearing was rigidly attached within the cuvette, and then from an oscillogram showing the changes in the torque at the axis of the cuvette, with the latter in rotation, we determined the form of $F_2(\varphi)$. It was found that

$$F_2(\varphi)=F_0(I)\sin(N\varphi), \tag{8}$$

where $F_0(I)$ is a function which depends on the force of the current within the magnetic circuit (the current within the circuit changes).

Substitution of relations (7) and (8) into (1) and the changeover to dimensionless variables on the basis of the formulas

$$\tau = t\,(k_1/m)^{1/(2-p)} R_1^{(p-1)/(2-p)}; \quad \omega_{*1} = d\varphi_*/d\tau,\ \omega_{*2} = \tau;$$
$$\Omega = \omega_0 (m/k_1)^{1/(2-p)} R_1^{(1-p)/(2-p)}; \quad F = F_0 (m^p/k_1^2 R_1^p)^{1/(2-p)}$$

lead to the following system:

$$d\omega_{*1}/d\varphi_* = [F \sin (N\varphi_*) - (|\omega_{*1} - \Omega|)^{p-1} (\omega_{*1} - \Omega)]/\omega_{*1};$$
$$d\omega_{*2}/d\varphi_* = \omega_{*1}^{-1}. \tag{9}$$

We examine the problem in which the "bearing-fluid" system, rotating at a velocity Ω, was subjected to a magnetic field at the instant at which it passed point $\varphi_* = 0$; in this case

$$\omega_{*1}(0) = \Omega; \quad \omega_{*2}(0) = 0. \tag{10}$$

As a result we found the periodic solution of system (9) and from this we determined the time by which the bearing lagged within a single revolution, and the increase in the torque on the shaft of the cuvette.

It follows from qualitative considerations that under conditions in which this periodic external force is effective the bearing is subjected to a oscillating-translational motion in which it gradually lags behind the fluid. In this case, additional dissipation of the mechanical energy is essential, and in the described experiments this occurs as the torque on the shaft of the cuvette is increased. This torque increment is determined from solution of problem (9) and (10). Some calculation results are presented in Fig. 4. With small amplitudes for the magnetic-field force F the change in the moment M is quite insignificant. However, on reaching some critical value there occurs a decisive increase in the torque. At the same time, the period $\overline{T}$ of bearing rotation begins also intensively to increase (Fig. 5). The increased dissipation of the mechanical energy due to the relative oscillational motion of the particle can be interpreted as the appearance of some additional viscosity, which may be identified as oscillational viscosity.

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