HYDRAULIC RESISTANCE IN THE FLOW OF AN ELECTRICALLY CONDUCTIVE FLUID IN TUBES IN A LONGITUDINAL MAGNETIC FIELD

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Analysis of the empirical formulas proposed by various authors for the hydraulic resistance in the flow of an electrically conductive fluid in tubes in a longitudinal magnetic field demonstrates that with expansion of the range of Reynolds and Hartmann numbers there is a transition from the decisive parameter Ha/R to a parameter that is close to the Stewart number $St = Ha^2/R$. In using the parameter Ha/R_* (where $R_* = V_* d/v$), determining the velocity profile, we find that the experimental data over the entire range of R and Ha numbers group themselves satisfactorily about a single curve.

On the basis of experimental studies of the hydraulic resistance in the flow of a liquid metal through tubes in a longitudinal magnetic field, various authors have proposed empirical relationships, presented here to expand the area of their application:

$$\lambda_H/\lambda = 1 - 185(\mathrm{Ha}/R)^{1.6}, \quad (R \leqslant 3\cdot 10^4, \quad \mathrm{Ha} \leqslant 146, \ [1]); \tag{1}$$

$$\frac{\lambda_H}{\lambda} = 1 - 37.7\frac{\mathrm{Ha}^{1.65}}{R^{1.45}} = 1 - 37.7\left(\frac{\mathrm{Ha}^{1.14}}{R}\right)^{1.45}, \quad (R \leqslant 5\cdot 10^4, \ \mathrm{Ha} \leqslant 300, [2]); \tag{2}$$

$$\lambda_H/\lambda = 1 - \mathrm{Ha}^{1.8}/R, \quad (R \leqslant 9\cdot 10^4, \quad \mathrm{Ha} \leqslant 1.5\cdot 10^3, [3]). \tag{3}$$

With an expansion in the range of Reynolds numbers R and Hartmann numbers Ha in these formulas, the exponent m in the decisive parameter Ha^m/R increases, i.e., there is a trend toward the transition from the parameter Ha/R for small R numbers to a parameter close to the Stewart number $St = Ha^2/R$ for large Reynolds numbers. This is easily understood if we bear in mind that there is a drop in the relative role of the viscosity region near the wall with an increase in R when the velocity profile is full, while outside of this region the flow characteristics are independent of viscosity. In the absence of a field, for large R numbers, the dependence of the resistance factor on the viscosity (i.e., on the R number) is preserved, since the friction at the wall is associated with the characteristics of the viscous region. For flow in a magnetic field the resistance factor for large R must also be a function of the viscosity. However, this relationship, as in the case without a field, must be relatively weak.

To describe the experimental data we can select a variety of formulas that are convenient for practical utilization. Thus, for example, reference [4] proposed the following formula:

$$\frac{\lambda - \lambda_H}{\lambda_H - \lambda_{1am}} = \frac{0.173X^{2.24}}{1 + 0.173X^{2.24}}, \quad X = \frac{\mathrm{Ha}}{0.1(R - 2000)^{0.77}}, \tag{4}$$

and this formula is valid over a wide range of R and Ha numbers.

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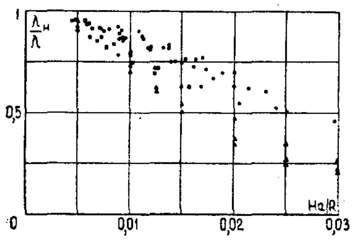

Fig. 1

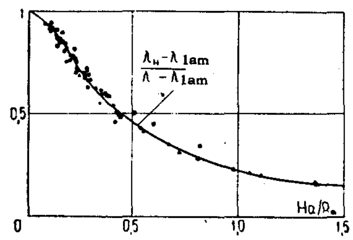

Fig. 2

For further clarification of the quantitative relationships governing flow it is a good idea to present the experimental data as a function of the parameter determining flow structure. It was demonstrated in [5, 6] that near the wall, i.e., for $\tau \cong \tau_w$ and $l \cong \varkappa y$, the universal velocity profile $\varphi(\eta)$ for flow in a longitudinal magnetic field depends on the single parameter Ha/R_*, i.e., $\varphi = \varphi(\eta, Ha/R_*)$, where $R_* = V_* d/\nu$, $V_* = \sqrt{\tau/\rho}$. The theoretical [6] and experimental [4] velocity profiles are qualitatively in good agreement with each other. The experiments have shown that the velocity profiles are satisfactorily described by the parameter Ha/R_*, not only near the wall, but throughout the entire tube cross section. This result is analogous to the known fact of approximate validity for the entire cross section of the logarithmic "law of the wall" and can be explained by the mutual compensation of the two effects: the reduction in the frictional stress τ with decreasing distance from the wall and the weakening of the growth in the scale l. The velocity-profile parameter Ha/R_* must approximately describe the experimental data on the hydraulic resistance over the entire range of R numbers.

Figure 1 shows the experimental data derived by various authors as functions of Ha/R; the same data are presented in Fig. 2 as functions of Ha/R_*. In using the parameter Ha/R_*, we find that the experimental data over the entire investigated range of R and Ha numbers satisfactorily group themselves about a single curve. Bearing in mind that with an increase, in the most favorable case, of the effect of total laminarization and flow stabilization, (i.e., $\lambda_H = \lambda_{lam}$, along the axis of ordinates), in Fig. 2 we plot the quantity $(\lambda_H - \lambda_{lam})/(\lambda - \lambda_{lam})$, where the subscript H refers to the flow in the magnetic field and the subscript lam refers to the stabilized laminar flow; λ is the resistance factor in the absence of a field.

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