INTERNAL HYDRAULICS OF MAGNETOHYDRODYNAMIC MACHINES WITH A NONHOMOGENEOUS DISTRIBUTION OF THE FORCES

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It is shown that if a moving electrodynamic force is distributed nonhomogeneously over the cross section of a channel, the pressure developing with turbulent flow must be determined by solution of the problem of the internal hydraulics of the channel. The possibility of such a solution is demonstrated with large values of the local slips. An analysis is made of the differences between the p(Q) characteristics obtained from the generally accepted characteristics. It is noted that the local velocities may strongly exceed the mean mass flow rate.

Existing methods for calculating the p(Q) characteristics of electromagnetic pumps, throttles, etc. are carried out in two stages. There is first calculated some arbitrary electrodynamic pressure p_0, determined as the ratio of the total electrodynamic force to the cross-section area of the channel, and then, to determine the pressure developed by the device p, p_0 is used to calculate the internal hydraulic losses.

$$p=p_0-\zeta Q^2 \operatorname{sign} Q. \tag{1}$$

In the simplest case, it is assumed that these losses are equal to the losses with flow outside of the field. In the most perfected methods, the coefficient ζ depends in one way or another on the determining criteria, as a rule set up on the basis of averaged values. Expression (1) means that a magnetohydrodynamic machine consists of an ideal source of pressure, p_0, and a hydraulic resistance $\zeta(Q)$ connected in series (Fig. 1, scheme a). For a magnetohydrodynamic device with a homogeneous distribution of the electrodynamic forces over the cross section of the channel, such a representation raises no objections. However, in the general case of nonohomogeneous distribution of the forces the picture of the flow in the channel becomes complex. Convincing evidence on this question is contained in the work of Okhremenko [1]. With a nonhomogeneous distribution of the forces, scheme "a" can be applied only to each elementary stream filament, within the limits of which the electromagnetic force may be assumed constant. The device as a whole may be regarded as connection in parallel of many such elementary stream filaments (Fig. 1, scheme b). Naturally, this type of idealized model cannot pretend to be a complete reproduction of the whole complex picture of the flow in a real pump. The model takes account only of the flow parameters which are responsible for the hydraulic losses. If we are interested only in the pressure developed by the device, i.e., the difference in the pressures between cross sections A and B, then with laminar flow ($\zeta(Q)\sim|Q|^{-1}$) scheme b is equivalent to scheme a if it is assumed that $p_0=\Sigma p_{0i}/n$. However, for turbulent flow (a quadratic resistance law), these schemes

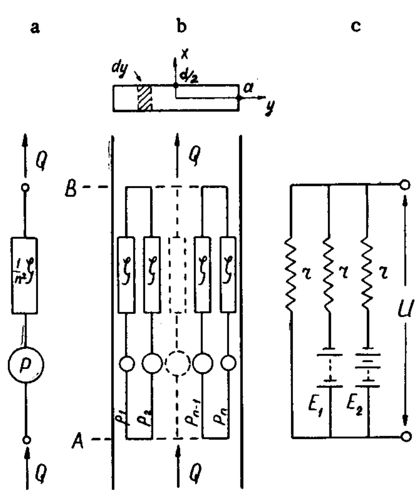

Fig. 1. Scheme for calculation of p(Q) characteristics.

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differ in principle. In this case, the pressure developed by the device depends to a considerable degree on the distribution of the electrodynamic force over the cross section of the channel. For clarity, let us examine the scheme c, which is analogous to the hydraulic model considered. If we introduce into the consideration a quadratic resistance law $U = ri^2 \operatorname{sign} i$, and if we use the Kirchhoff laws, we obtain

$$U = |U-E_1| + |U-E_2| + \\ + 2\sqrt{|U-E_1||U-E_2|} \operatorname{sign}(E_1-U) \operatorname{sign}(E_2-U). \qquad (2)$$

If we set $E_1 = D_2$, then $U = 1.2 \cdot {}^1\!/_3(E_1 + E_2)$; if $E_1 = 2E_2$, then $U = {}^1\!/_3(E_1 + E_2)$; if $E_2 = 0$, then $U = 0.6 \cdot {}^1\!/_3(E_1 + E_2)$. Thus, depending on the distribution of the electrodynamic forces, the voltage U in which we are interested may be greater than, equal to, or less than the mean arithmetical value of the emf's under consideration. It is clear that U depends also on the distribution of the resistances r. Scheme c is analogous for a device with a closed channel. We arrive at the conclusion that the pressure developed by the device under such conditions in the general case must differ from the above-discussed electrodynamic pressure p_0. Since the above considerations must also be qualitatively valid in the case of a transit flow rate Q, it can be asserted that calculation of the pressure using an expression with the structure (1) is physically improper if the electrodynamic force over the cross section of the channel is nonhomogeneous. In this case, the pressure developed by the device must be determined by solution of the problem of the internal hydraulics of the device. This leads to a more correct calculation of the quadratic character of the turbulent flow. A similar problem for a channel with a rotating magnetic field was investigated by Yantovskii [2, 3]. Below the possibility of such a calculation is demonstrated using the example of a flat channel without busbars, located in a running field in a section of finite length. In this case, as solutions of the corresponding electrodynamic problem, we use the results of the well-known work of Vol'dek [4].

We assume that an ideal running field acts in a section of length l (Fig. 1b). If the hydraulic phenomena in the transitional regions around the cross sections A and B are neglected, the balance of forces in the active section may be written in the form

$$\frac{\lambda\rho}{2d} v^2(y) \operatorname{sign} v(y) = f(y) - \frac{p}{l}. \qquad (3)$$

The term in the left-hand part of Eq. (3) expresses the force of friction, where λ is the friction coefficient, ρ is the density of the liquid, d is the thickness of the slot-type channel, and v (y) is the mean velocity of the liquid over the thickness of the channel in a cross section with the coordinate y. Writing the force of friction in this form means that account is taken only of friction on the long walls of a cross section of the channel; the dependence of the friction for each elementary stream filament, cut in a direction perpendicular to the long sides (Fig. 1), is assumed to be the same as for friction in an ordinary flat infinitely broad channel outside of the field. In other words, this is equivalent to the postulation that $\partial v/\partial x \gg \partial v/\partial y$. In the present work, the friction coefficient λ is assumed to be independent of the velocity.

f (y) denotes the density of the moving electrodynamic force in accordance with [4] depending on y.

$$f(y) = \frac{\omega}{2\alpha} \sigma B^2 \Re e \frac{\alpha^2}{\gamma^2}\left(1 - \frac{\operatorname{ch} \gamma y}{\operatorname{ch} \gamma a}\right); \qquad (4)$$

$\alpha = \pi/\tau$, $\gamma^2 = \alpha^2(1 + i\varepsilon)$, $\varepsilon = \mu_0 \sigma\omega\tau^2/\pi^2$, ω is the angular frequency, σ is the electrical conductivity, B is the amplitude of the magnetic induction, and τ is the polar scaling of the inductor. The assumption that f (y) does not depend on v (y) means that we are considering only the case of a large slip, i.e., $s(y) \approx 1$ for all values of y. This condition limits the generality of the quantitative results obtained. However, it is interesting with respect to the fact that all the effects established are due to purely hydraulic processes with turbulent flow in a nonhomogeneous force field.

By p there is denoted the pressure drop in the section AB. The neglect of processes in the transitional regions around the cross sections A and B means that the results obtained are applicable only for sections of sufficient length.

On the basis of (2) we can write an expression for the mass flow rate.

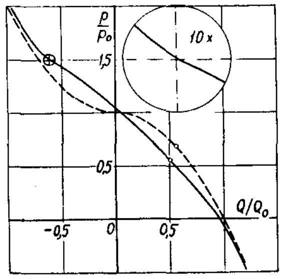

Fig. 2. p (Q) Characteristic at $a \ll \tau$.

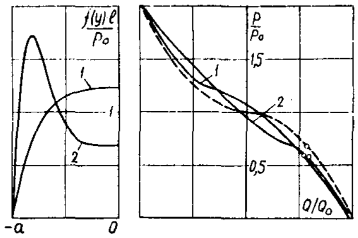

Fig. 3. p (Q) Characteristics and curves of the force for the cases $a = \tau$, $\varepsilon = 1$ (curve 1) and $a = \tau$, $\varepsilon = 6$ (curve 2).

$$Q(p) = 2d\int_0^a v(y)\,dy = \sqrt{\frac{8d^3}{\lambda\rho}}\int_0^a \operatorname{sign}\left(f(y) - \frac{p}{l}\right)\sqrt{f(y) - \frac{p}{l}}\,dy. \tag{5}$$

The result of calculation of the integral (5) is conveniently presented in the coordinates $p/p_0 = \psi(Q/Q_0)$. We recall that p_0 denotes the electrodynamic "pressure," calculated in accordance with [4].

$$p_0 = \frac{l}{a}\int_0^a f(y)\,dy = \frac{\omega}{2\pi}\,l\tau\sigma B^2\Re\frac{\alpha^2}{\gamma^2}\left(1 - \frac{\operatorname{th}\gamma a}{\gamma a}\right). \tag{6}$$

Q_0 is the mass flow rate in the channel outside of the field, corresponding to the pressure drop p_0, i.e.,

$$Q_0 = \sqrt{\frac{8a^2d^3p_0}{\lambda\rho l}}. \tag{7}$$

In the limiting case $a \ll \tau$, integral (5) can be calculated analytically, which is of interest for study of the properties of the dependence $p/p_0 = (Q/Q_0)$. Then

$$f(y) = \frac{3}{2}\frac{p_0}{l}\left[1 - \left(\frac{y}{a}\right)^2\right] \tag{8}$$

and the integral is expressed by the three formulas

$$\frac{Q(p)}{Q_0} = -\frac{1}{2}\sqrt{\frac{p}{p_0}} - \frac{1}{\sqrt{6}}\left(\frac{p}{p_0} - \frac{3}{2}\right)\ln\frac{\sqrt{3/2} + \sqrt{p/p_0}}{\sqrt{p/p_0 - 3/2}} \quad \text{at} \quad \frac{p}{p_0} > \frac{3}{2},$$

$$\frac{Q(p)}{Q_0} = -\frac{1}{2}\sqrt{\frac{p}{p_0}} - \frac{1}{\sqrt{6}}\left(\frac{p}{p_0} - \frac{3}{2}\right)\left[\ln\frac{\sqrt{3/2} + \sqrt{p/p_0}}{\sqrt{3/2 - p/p_0}} + \frac{\pi}{2}\right] \quad \text{at} \quad 0 < \frac{p}{p_0} < \frac{3}{2},$$

$$\frac{Q(p)}{Q_0} = \frac{1}{2}\sqrt{\frac{|p|}{p_0}} - \frac{1}{\sqrt{6}}\left(\frac{p}{p_0} - \frac{3}{2}\right)\arcsin\left(1\Big/\sqrt{1 - \frac{2p}{3p_0}}\right) \tag{9}$$

$$\text{at} \quad p < 0.$$

Figure 2 gives the p (Q) characteristic, constructed using formulas (9) (solid line). For purposes of comparison, the dotted line on the same figure gives a plot of the curve of $p/p_0 = 1 - \operatorname{sign} Q\,(Q/Q_0)^2$, expressing graphically the simplest dependence (1).

It is evident that, under both pumping and braking conditions, the curves differ appreciably. Let us take note of some of the characteristic special features of this difference.

The solid curve intersects the p axis somewhat above the dotted curve. This means that, under the given conditions, a closed channel develops a static pressure 4.8% greater than p_0. The maximal hydraulic power $(pQ)_{max}$ is 72.9% of that calculated using (1) and corresponds approximately to the same values of Q/Q_0 (small circles on curves).

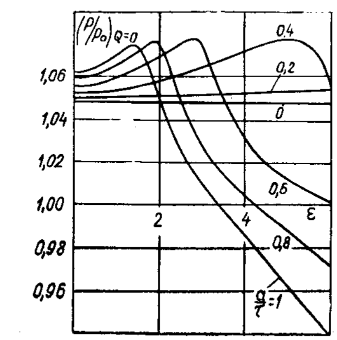

Fig. 4. Pressure in a closed channel as a function of a/τ and ε.

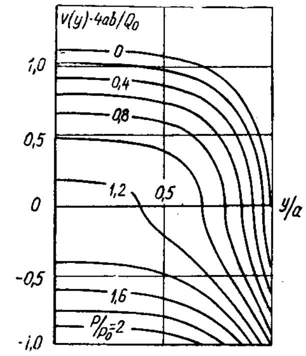

Fig. 5. Distribution of velocities for $a = \tau$ and $\varepsilon = 1$.

In distinction to the dotted curve, the solid curve intersects the p axis at an oblique angle and at the point of intersection is not characteristic for a first point of inflection with a horizontal tangent. The solid curve also has such a point of inflection at $p = {}^3/_2 p_0$; however, it is less sharply expressed than for the dotted line, which has a branching point of the type of a square root, while the solid curve has a logarithmic point. Mathematically, at the point of inflection, the solid line also has a horizontal tangent; however, the accuracy of the figure does not permit showing this. Even with a tenfold magnification, the point of inflection appears merely as a break. In addition to the point $p = {}^3/_2 p_0$, the solid curve has a second singular point, i.e., a discontinuity of the second derivative at $p = 0$.

Physically, both singular points of the solid curve are due to changes in the distribution of the velocities over the width of the channel, taking place at pressures of $p = 0$ and $p = {}^3/_2 p_0$. In the interval $0 < p < {}^3/_2 p_0$, there exist mutually opposing flows in the channel, while, at $p < 0$, all the velocities are in a forward direction and, at $p > {}^3/_2 p_0$, only in the reverse direction. At the point $p = 0$, the singularity is weakly expressed, since this point corresponds to the disappearance of the reverse flow in a narrow region along the edges of the channel. With an approach to the point $p = {}^3/_2 p_0$, the forward flow at the center of the channel vanishes. Since the curve of the forces in this region of the channel is relatively flatter, this involves a considerably greater mass of liquid. The singularity of the dashed line at the point $p = p_0$ is expressed to an even more marked degree, since (1) assumes identical values of the velocities over the whole breadth and, by virtue of this, with passage through the point $p = p_0$, there is a change in the direction of the flow over the whole breadth of the channel.

The limiting case considered $a \ll \tau$, is mainly of methodological rather than practical interest. For a number of values of the parameters ε and a/τ, which are characteristic for industrial practice, the integral (5) was calculated in a computer. Figure 3 gives two typical p(Q) characteristics. On the left-hand side of this figure, there are given the corresponding curves of the force $f(y)$; the values of the quantity $f(y) l/p_0$ are plotted along the axis of ordinates. This makes it possible to say that the extrema of the curve are located on the same ordinate as the corresponding point of inflection on the p(Q) characteristic. The above-mentioned regularity is observed: if the extremum is located on the flatter part of the curve of the forces, the point of inflection on the p(Q) characteristic is more strongly expressed.

For all the curves calculated, the following is characteristic. a) if the curve of $f(y)$ has no minima, the pressure developed at $Q = 0$ is somewhat greater than p_0. In the presence of minima, at $Q = 0$, p may also be less than p_0. The dependence of $p_{Q=0}$ on the parameters a/τ and ε is shown in Fig. 4. b) The extrema on the curves of $f(y)$ correspond to points of inflection on the p(Q) characteristics. c) Under the conditions $p = 0$, the mass flow is always a few percent less than Q_0.

Special emphasis must be laid on the question of the value of the internal velocities. Figure 5 shows the distribution of the velocities over the width of the channel for $a = \tau$ and $\varepsilon = 1$. The parameter for the curves is the ratio p/p_0. It is evident that even in a closed channel ($p/p_0 \approx 1.06$) the local velocity attains values of $0.4 Q_0/2ad$ at the center and values of $\sim Q_0/2ad$ at the edges of the channel.

The presence in the channel of velocities which are capable of considerably exceeding the mean velocity forces us, in particular, to consider the possibility that in the channels under consideration there may suddenly and unexpectedly arise phenomena of a cavitational character.

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