TWO-DIMENSIONAL TURBULENT FLOW IN A CHANNEL WITH INHOMOGENEOUS ELECTRICAL CONDUCTIVITY OF THE WALLS

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An investigation of an analog of Lehnert flow in a prismatic tube of prismatic cross section was carried out. It is shown experimentally that in a sufficiently strong magnetic field the mean and fluctuation structures of the flow acquire a sharply expressed two-dimensional character.

Since the appearance of Lehnert's papers [1, 2] it has been known that a magnetic field not only increases the stability of flows of conducting fluids, but also under specific conditions it is capable of leading to destabilization of the flow. The basic, and probably the only reason for the destabilization, is the deformation of the mean flow, causing the appearance of inflection points in the velocity profile. The presence of these points also leads to a decrease in the stability of the flow as well as the generation of perturbations of considerable intensity.

One should bear in mind that the loss of stability of the flow on account of the mechanism indicated is accompanied by the stabilizing action of the field directly on the perturbation. If the boundaries of flow are nonconducting, this last effect becomes less important as the magnetic field is increased because of the rearrangement of the flow into a plane flow in the plane perpendicular to the field. On such a flow, as is well-known, the magnetic field has no effect, and consequently, it exerts no action on the perturbations in the plane perpendicular to the field.

Highly unstable flows in a magnetic field develop, for example, when there is inhomogeneity in the electrical conductivity of their boundaries. Both theoretical investigations of laminar flows of this kind [3] and experimental studies of actual flows [4-8] are well-known.

Recently a direct measurement was made of the intensity of perturbations in a flow of Hunt's type [9] and in a flow with large velocity gradients at a distance from the solid boundaries [10]. For both these experiments one common property was characteristic: for large Hartmann numbers the flow was concentrated in narrow zones of thickness $\mathrm{Ha}^{-1/2}$ with a maximum velocity of order Ha. This indicates that for such flows the characteristic Reynolds number increases like $\mathrm{Ha}^{1/2}$. An increase in the magnetic field must lead to a considerably slower supression of the generation of three-dimensional perturbations, since laminar flow* must begin considerably later (i.e., for exceedingly large ratios Ha/Re) than for ordinary Hartmann flow. Nevertheless, satisfactory agreement in the magnitudes of the drag coefficient at large values of the Hartmann number, obtained in [11, 12] experimentally for an actual flow and theoretically for a laminar flow, and at the same time the presence in such a flow of fluctuations of high intensity (25-40%) [9, 10] indicates that with an increase in the magnetic field there appears a tendency toward two-dimensional structure of the turbulence. This is also indicated by the decrease with an increase in the field of the ratios $\tilde{e}'_y/\tilde{e}'_z$

* Here the term laminarization is understood to mean the matching of the drag coefficients of laminar and the actual flow. On the basis of results that are already well-known, it can be stated that in the flows under consideration laminarization, in the sense of supression of fluctuations, etc., does not occur.

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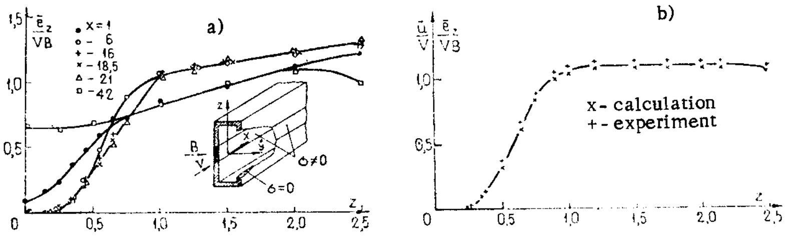

Fig. 1. a) Experimental segment of the channel and distribution of the z component of the constant electric field with height in the channel for y = 0 and various values of x, B = 0.8 T; b) results of numerical calculation of the longitudinal velocity and the z component of the constant electric field. Ha = 100. The wall of the channel corresponds to z = 3.

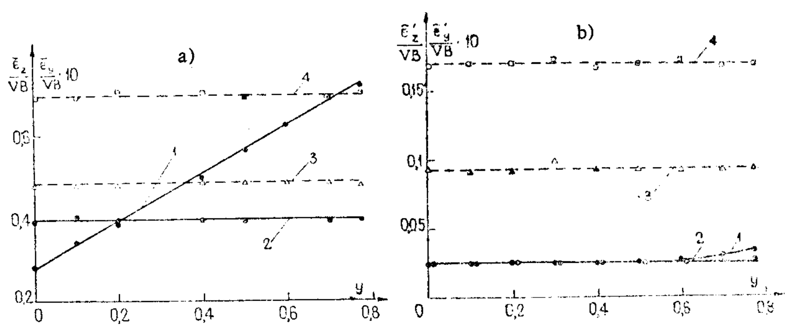

Fig. 2. Distribution of the constant (a) and the fluctuating (b) components of the electric field along the direction of the magnetic field for z = 0.5. The solid lines are the y component, and the dashed lines are the z component. The dependences 1 and 3 correspond to x = 10, and 2 and 4, to x = 42. B = 0.8 T.

and $\tilde{e}'_y/\tilde{e}'_x$ (where $\tilde{e}'_y$ is the intensity of the component of the electric field parallel to the field, i.e., the fluctuation intensity of the y component of the electric current).

In [13] the question concerning various possibilities of realizing two-dimensional turbulent flows by means of strong magnetic fields was discussed, in particular, flows with an M-shaped velocity profile.

From the point of view of the author of the present paper, one of the possibilities of realizing such a flow is the analogue of a Lehnert flow in a prismatic tube with rectangular cross section (Fig. 1a). Such a flow differs from those cited above first of all in that the maximum velocity in it is bounded above by a quantity not depending on the magnitude of the Hartmann number, which is determined only by the ratio of the areas of the conducting and nonconducting parts of the channel walls.

Below are given the results of an investigation of the mean and fluctuation characteristics of the flow cited above in a strong magnetic field.

1. The experiment was carried out on a horizontal mercury circuit with a constant-current conduction pump [14]. The operative channel of length 1200 mm with a 20×60 mm^2 cross section was placed in the 40 mm gap of an electromagnet with 90×900 mm^2 poles, producing a maximum induction B = 1.1 T.

Two copper strips, mounted in the channel (Fig. 1a), had a length of 260 mm, a width of 10 mm, and a thickness of 2 mm. The measurements were made with a conduction anemometer with a three-electrode pick-up at various distances from the front edge of the strips under conditions of a mean flow speed V = 20 cm/sec and values 0.08 and 0.8 T of the induction B of the magnetic field.

For the measurement of the magnitude of the component of the constant electric field a type F116 microvoltammeter was employed. To obtain the fluctuation characteristics of the flow an amplifier, having a pass band from 1 Hz to 10 kHz and a voltmeter, giving root-mean-square readings, manufactured by the Disa Electronic firm was used. Spectral analysis was performed by an NCh-type FSP-80 spectrum analyzer, manufactured by the RFT firm.

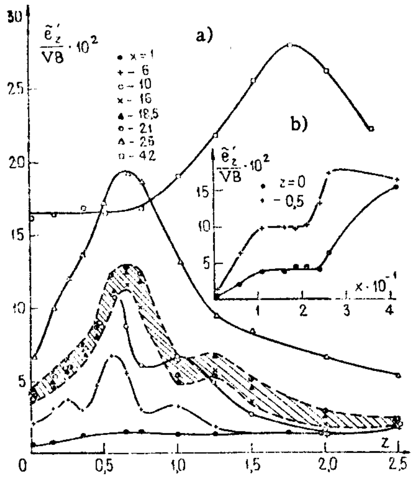

Fig. 3

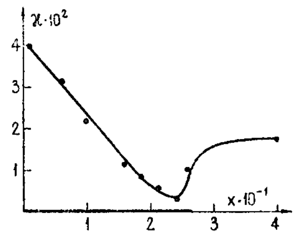

Fig. 4

Fig. 3. Distribution of fluctuations of the electric field: a) z component as a function of height in the channel; b) z component along the length of the channel. y = 0, B = 0.8 T.

Fig. 4. Dependence of the coefficient of three-dimensionality on the coordinate along the flow direction, y = 0, z = 0.5.

2. As shown in [15], at large Hartmann numbers the components $\overline{e}_z/VB$ and $\tilde{e}'_z/VB$ of the constant and variable electric fields, respectively, coincide very exactly with the longitudinal velocity $\overline{u}/V$ and the intensity of the longitudinal velocity fluctuations $\tilde{u}'/V$. For the mean quantities this correspondence is confirmed by numerical calculation of a one-dimensional flow with a conducting strip (Fig. 1b; here and below the width of the strip is taken as the characteristic dimension), performed especially in order to compare the quantities $\overline{e}_z/VB$ and $\overline{u}/V$. The method of calculation is described in [16].

Results of measurements of the constant z component of the electric field at various distances from the front edge of the copper strip are shown in Fig. 1a. As is evident from the figure, in the region adjacent to the conducting part of the wall the fluid is completely braked, so that the velocity profile has a sharply delineated M-shaped character. The profiles of $\overline{e}_z/V$ and $\overline{e}_y/V$ in their dependence on the coordinate in the direction of the field are shown in Fig. 2a. These profiles clearly demonstrate the two-dimensionality of the flow, i.e., the lack of dependence of the mean characteristics on the coordinate y. The linear dependence of $\overline{e}_y/VB$ on y in the region of the copper strip indicates that the y component of the curl of the electromagnetic force, proportional to the quantity $\partial j_y/\partial y$, does not depend on y, and consequently that the y component of the curl of the velocity does not depend on y either [5].

In consequence of the fact that a magnetic field does not act on such a plane flow, the flow that we are considering, similar to a flow with an M-shaped velocity profile with the field absent, is unstable and capable of generating perturbations of considerable intensity. In fact, in our experiments, as was the case in those of Lehnert, perturbations of great intensity were observed.

Results of measurements of the z component of fluctuations of the electric field, shown in Fig. 3, show how the intensity of the velocity fluctuations begins to grow rapidly with increasing distance from the front edge of the strip within the limits of the region of the conducting part of the channel wall, and then, starting at a distance x = 10, tends to some asymptotic distribution and only in the immediate vicinity of the trailing edge of the strip, where there is an abrupt change in the conductivity of the wall, it increases once more (Fig. 3a). Note that the distribution of the intensity of the perturbations along z (Fig. 3b) is extremely inhomogeneous with a pronounced maximum located, as might be expected, in the neighborhood of the inflection point of the mean velocity profile.

As Fig. 2b implies, the intensity of the perturbations is practically independent of y. Moreover the quantity $\tilde{e}'_y/VB$ is an order of magnitude smaller than $\tilde{e}'_z/VB$. All this indicates that the fluctuation characteristics of the flow also have a pronounced two-dimensional character.

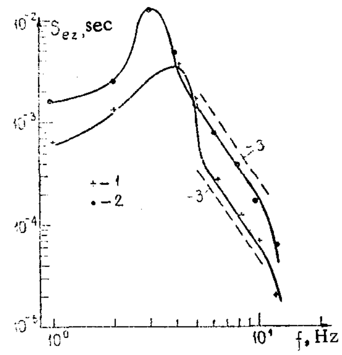

Fig. 5

Fig. 6

Fig. 5. Spectra of fluctuations of the z component of the electric field. Curve 1 for x = 10, 2 for x = 42. z = 0.5, B = 0.8 T.

Fig. 6. Oscillograms of fluctuations of the z component of the electric field. a) x = 10; b) x = 42. B = 0.8 T, z = 0.5.

In order to estimate the degree of three-dimensionality of the structure of the perturbations it is convenient to use the coefficient of three-dimensionality $\varkappa = 2e'_y{}^2/e'_z{}^2$ [14]. The variation of $\varkappa$ in the direction of the flow is shown in Fig. 4. From the figure one sees that in a sufficiently strong magnetic field the coefficient $\varkappa$ is of order $\sim 10^{-2}$, while in a weak magnetic field $\varkappa = 0.9-1.12$.

Results of a spectral analysis (Fig. 5) also indicate the realization of two-dimensional turbulence. As is seen from the figure, an inertial interval with the −3 law, characteristic of two-dimensional turbulence, appears in the spectra, which characterizes the absence of transferral of energy to the upper part of the spectrum. The displacement of the spectrum to the left (x = 42) indicates the transferral of energy to the lower part of the spectrum, i.e., from higher to lower frequencies.

Note that the intensity of the perturbations sufficiently far downstream from the conducting part is considerably higher than it is in the region of the conducting part (Fig. 3). This is evidently explained by the fact that the perturbations near the conducting part of the channel wall dissipate with greater intensity than those in sections beyond the limits of the conducting strips.

Figure 6 shows examples of oscillograms of fluctuations of the z component of the electric field, which indicate that within the limits of the copper strip (x = 10) the perturbations have a rather regular character. This is in good agreement with results [2] of Lehnert, who visually observed a system of regular vortices in his experiments. As they move downstream, the perturbations lose their regularity, while at the same time their frequency becomes somewhat lower than that of the perturbations within the limits of the copper strip.

It has been shown above that in a strong magnetic field, with a specially selected inhomogeneity in the electrical conductivity of the channel walls, a two-dimensional turbulent flow with an M-shaped velocity profile can be realized.

In contrast to the flow behind a circular cylinder whose axis is parallel to the magnetic field [14], in the flow described above generation of the two-dimensional perturbations is effected by the M-shaped velocity profile with inflection points alone. In flow about a cylinder in the stream, in addition to the perturbations generated by a similar velocity profile, perturbations generated directly by the cylinder were present.

As in the case of flow behind a circular cylinder [14], in the spectrum of the perturbations there was an inertal interval with the −3 law, characteristic of two-dimensional turbulence, and a transfer of energy from perturbations with small scales to perturbations with large scales also occurred.

In conclusion I thank A. B. Tsinober for guidance and help in carrying out the work, and also G. Vitolin'sh for performing the calculation of the flow discussed in the paper (Fig. 1b).

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